Find the current as a function of time.

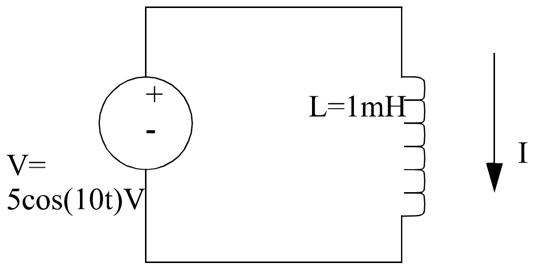
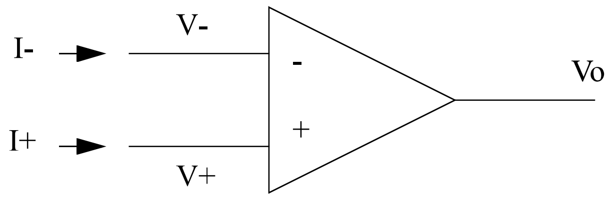


Figure 184 Drill problem: Find the current through the inductor

7.2.5 Op-Amps

The ideal model of an op-amp is shown in Figure 185. On the left hand side are the inverting and non-inverting inputs. Both of these inputs are assumed to have infinite impedance, and so no current will flow. Op-amp application circuits are designed so that the inverting and non-inverting inputs are driven to the same voltage level. The output of the op-amp is shown on the right. In circuits op-amps are used with feedback to perform standard operations such as those listed below.

- adders, subtractors, multipliers, and dividers - simple analog math operations
- amplifiers - increase the amplitude of a signal
- impedance isolators - hide the resistance of a circuit while passing a voltage



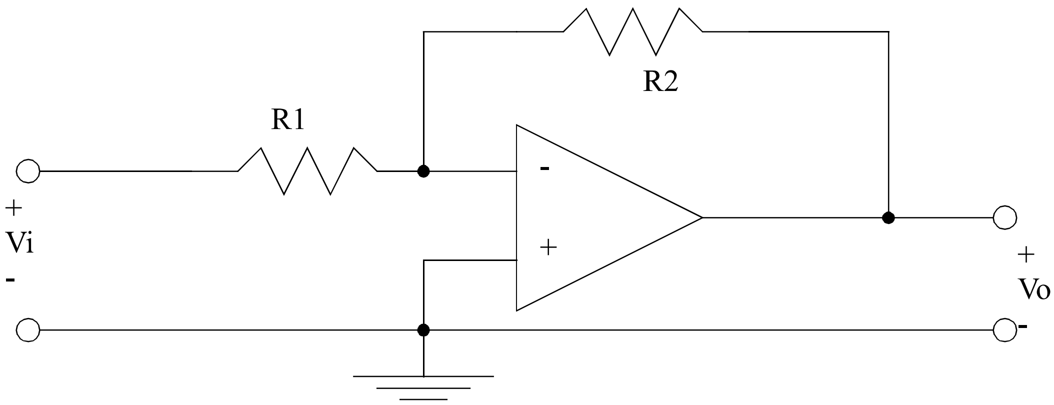
Note: for analysis use,

$$I_- = I_+ = 0$$

$$V_- = V_+$$

Figure 185 An ideal op-amp

A simple op-amp example is given in Figure 186. As expected the voltages on both of the op-amp inputs are the same. This is a function of the circuit design. (Note: most op-amp circuits are designed to force both inputs to have the same voltage, so it is always reasonable to assume they are the same.) The non-inverting input is connected directly to ground, so it will force both of the inputs to 0V. When the currents are summed at the inverting input, an equation with both the input and output voltages is obtained. The final equation shows the system is a simple multiplier, or amplifier. The gain of the amplifier is determined by the ratio of the input and feedback resistors.



The voltage at the non-inverting input will be 0V, by design the voltage at the inverting input will be the same.

$$V_+ = 0V$$

$$V_- = V_+ = 0V$$

The currents at the inverting input can be summed.

$$\sum I_{V-} = \frac{V_- - V_i}{R_1} + \frac{V_- - V_o}{R_2} = 0$$

$$\frac{0 - V_i}{R_1} + \frac{0 - V_o}{R_2} = 0$$

$$V_o = \frac{-R_2 V_i}{R_1}$$

$$V_o = \left(\frac{-R_2}{R_1}\right) V_i$$

Figure 186 A simple inverting operational amplifier configuration

An op-amp circuit that can subtract signals is shown in Figure 187.

Find the input/output ratio,

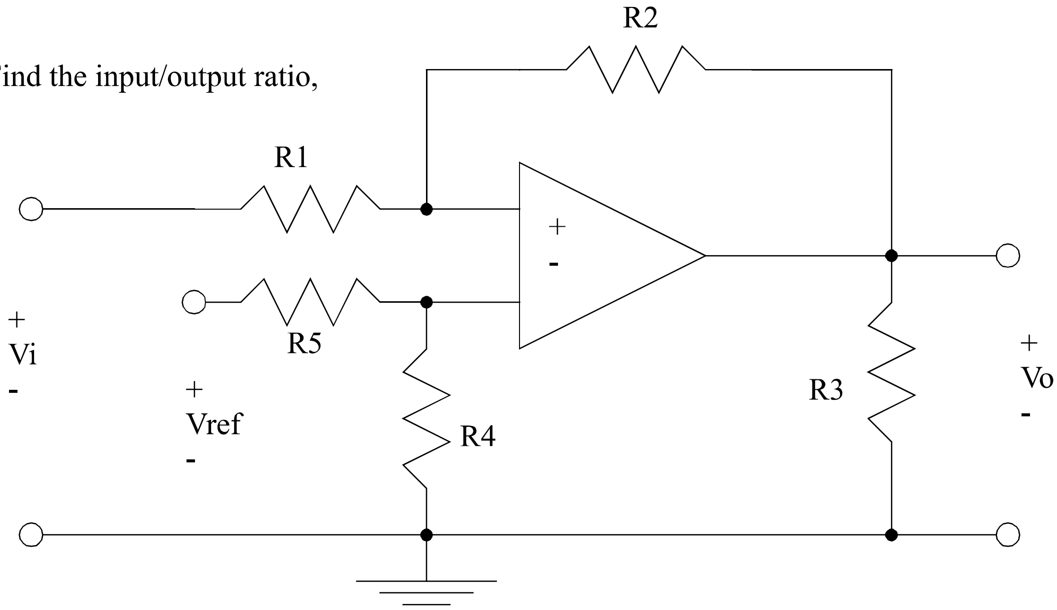


Figure 187 Op-amp example

For ideal op-amp problems the node voltage method is normally the best choice. The equations for the circuit in Figure 187 and derived in Figure 188. The general approach to this solution is to sum the currents into the inverting and non-inverting input nodes. Notice that the current into the op-amp is assumed to be zero. Both the inverting and non-inverting input voltages are then set to be equal. After that algebraic manipulation results in a final expression for the op-amp. Notice that if all of the resistor values are the same then the circuit becomes a simple subtractor.

Note: normally node voltage methods work best with op-amp circuits, although others can be used if the non-ideal op-amp model is used.

First sum the currents at the inverting and non-inverting op-amp terminals.

$$\sum I_{V+} = \frac{V_+ - V_i}{R_1} + \frac{V_+ - V_o}{R_2} = 0$$

$$V_+\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_i\left(\frac{1}{R_1}\right) + V_o\left(\frac{1}{R_2}\right)$$

$$V_+\left(\frac{R_1 + R_2}{R_1 R_2}\right) = V_i\left(\frac{1}{R_1}\right) + V_o\left(\frac{1}{R_2}\right)$$

$$V_+ = V_i\left(\frac{R_2}{R_1 + R_2}\right) + V_o\left(\frac{R_1}{R_1 + R_2}\right) \qquad (1)$$

$$\sum I_{V-} = \frac{V_- - V_{ref}}{R_5} + \frac{V_-}{R_4} = 0$$

$$V_-\left(\frac{1}{R_4} + \frac{1}{R_5}\right) = V_{ref}\left(\frac{1}{R_5}\right)$$

$$V_- = V_{ref}\left(\frac{R_4}{R_4 + R_5}\right) \qquad (2)$$

Now the equations can be combined.

$$V_- = V_+$$

$$V_{ref}\left(\frac{R_4}{R_4 + R_5}\right) = V_i\left(\frac{R_2}{R_1 + R_2}\right) + V_o\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_o\left(\frac{R_1}{R_1 + R_2}\right) = V_i\left(\frac{R_2}{R_1 + R_2}\right) - V_{ref}\left(\frac{R_4}{R_4 + R_5}\right)$$

$$V_o = V_i\left(\frac{R_2}{R_1}\right) - V_{ref}\left(\frac{R_4(R_1 + R_2)}{R_1(R_4 + R_5)}\right)$$

Figure 188 Op-amp example (continued)

An op-amp (operational amplifier) has an extremely high gain, typically 100,000 times. The gain is multiplied by the difference between the inverting and non-inverting terminals to form an output. A typical op-amp will work for signals from DC up to about

100KHz. When the op-amp is being used for high frequencies or large gains, the model of the op-amp in Figure 189 should be used. This model includes a large resistance between the inverting and non-inverting inputs. The voltage difference drives a dependent voltage source with a large gain. The output resistance will limit the maximum current that the device can produce.

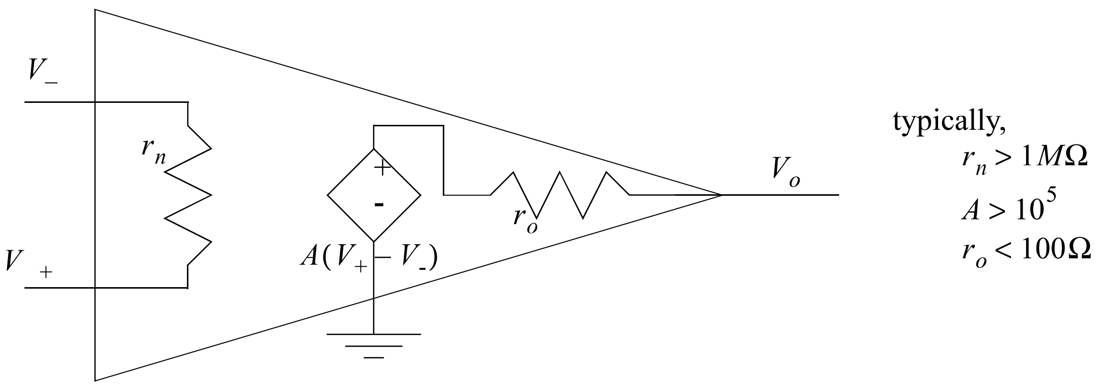


Figure 189 A non-ideal op-amp model

7.3 IMPEDANCE

Circuit components can be represented in impedance form as shown in Figure 190. When represented this way the circuit solutions can focus on impedances, 'Z', instead of resistances, 'R'. Notice that the primary difference is that the differential operator has been replaced. In this form we can use impedances as if they are resistances.

Device	Time domain	Impedance
Resistor	$V(t) = RI(t)$	$Z = R$
Capacitor	$V(t) = \frac{1}{C}\int I(t)dt$	$Z = \frac{1}{DC}$
Inductor	$V(t) = L\frac{d}{dt}I(t)$	$Z = LD$

Note: Impedance is like resistance, except that it includes time variant features also.

$$V = ZI$$

Figure 190 Impedances for electrical components